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The onset of Lapwood–Brinkman convection using a thermal non-equilibrium model

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Abstract

The stability of a horizontal fluid saturated sparsely packed porous layer heated from below and cooled form above when the solid and fluid phases are not in local thermal equilibrium is examined analytically. The Lapwood–Brinkman model is used for the momentum equation and a two-field model is used for energy equation each representing the solid and fluid phases separately. Although the inertia term is included in the general formulation, it does not affect the stability condition since the basic state is motionless. The linear stability theory is employed to obtain the condition for the onset of convection. The effect of thermal non-equilibrium on the onset of convection is discussed. It is shown that the results of Darcy model for the non-equilibrium case can be recovered in the limit as Darcy number $Da \rightarrow 0$. Asymptotic analysis for both small and large values of the inter phase heat transfer coefficient H is also presented. An excellent agreement is found between the exact solutions and asymptotic solutions when H is very small.

Keywords: Convection; Thermal non-equilibrium; Brinkman model; Porous medium

1. Introduction

The problem of convective instability of horizontal porous layer subject to an adverse temperature gradient has been investigated extensively by several authors in the past [1–5]. It is important to note that all the above-mentioned studies are based on the Darcy model. However, it is now realized that the Darcy model is applicable only under special circumstances and a generalized model for the accurate prediction of convection in a porous media must include Brinkman viscous term and Forchheimer inertia term. During the last decade, there has been a great upsurge of interest in determining the effects of extensions to Darcy's law since many practical applications involve media for which Darcy's law is inadequate. Some of the early works to deal with these extension are by Rudraiah et al. [6], Georgiadis and Catton [7], and Kladias and Prasad [8] who used the Darcy–Brinkman (DB) model and Darcy–Brinkman–Forchheimer (DBF) model for studying Benard convection in porous media. Many more works are available on the non-Darcy–Benard convection in a porous medium. The growing volume of work devoted to this area is well documented by the most recent reviews of Nield and Bejan [9] and Ingham and Pop [10].

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Nomenclature

c	specific heat
d	height of the porous layer
Da	Darcy number, $\frac{\mu_e K}{\mu_f d^2}$
$\mathbf{g}$	gravitational acceleration
h	inter phase heat transfer coefficient
H	non-dimensional inter phase heat transfer coefficient, $\frac{hd^2}{\varepsilon k_f}$
k	horizontal wave number
k_f, k_s	thermal conductivities of fluid phase and solid phase respectively
K	permeability of the porous medium
Pr	Prandtl number, $\frac{\mu_e(\rho c)_f}{\rho_0 k_f}$
p	pressure
$\mathbf{q}$	velocity vector, (u, v)
Ra	Rayleigh number, $\frac{\rho_0 g\beta(T_l - T_u)Kd}{\varepsilon\mu_f\kappa_f}$
T	temperature
t	time
(x, y)	space coordinates

Greek symbols

α	diffusivity ratio
β	coefficient of thermal expansion
γ	porosity-modified conductivity ratio, $\frac{\varepsilon k_f}{(1-\varepsilon)k_s}$
ε	porosity
κ	thermal diffusivity
μ_e	effective viscosity
μ_f	fluid viscosity
ρ_f	fluid density
ψ	non-dimensional stream function
θ	non-dimensional temperature of the fluid phase
ϕ	non-dimensional temperature of the solid phase
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Subscripts/superscripts

f	fluid
l	lower
s	solid
u	upper
*	non-dimensional
o	reference

Most of the works on convective heat transfer in porous media have mainly been investigated under the assumption that the fluid and the porous medium are everywhere in local thermodynamic equilibrium (LTE), although in many practical applications the solid and the fluid phases are not in thermal equilibrium. Nield and Bejan [9] have discussed a two-field model for energy equation. Instead of having a single energy equation, which describes the common temperature of the saturated media, two equations are used for fluid and solid phase separately. In the two-field model, the energy equations are coupled by the terms, which account for the heat lost to or gained from the other phase. Rees and co-workers [11–14] in a series of studies have investigated the non-equilibrium effect on free convective flows in porous media using Darcy model. The review by Kuznetsov [15] gives a detailed information about the works on thermal non-equilibrium effects.

In this paper we study the onset of convection in a sparsely packed porous layer heated from below with emphasis on how the condition for the onset of convection is modified when the solid and fluid phase are not in local thermal equilibrium (non-LTE). As discussed by Banu and Rees [12] when non-equilibrium effects are included in the problem the linear analysis is modified and it is still possible to proceed analytically to find the condition for the onset of convection. We have also carried out the asymptotic analysis for very small and very large values of the inter phase heat transfer coefficient. This work is more general than that of Banu and Rees [12] in the sense that we recover their results in the limit as the Darcy number Da tends to zero.

2. Mathematical formulation

We consider a Boussinesq fluid saturated porous layer of depth d, which is heated from below and cooled from above. A Cartesian coordinate system is chosen with the origin on the lower boundary and y-axis vertically upward. The lower surface is held at temperature T_l, while the upper surface is at T_u. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two-field model for temperatures. It is assumed that at the bounding surfaces the solid and fluid phases have identical temperatures. The Lapwood–Brinkman model is employed for the momentum equation. The basic governing equations are (see [9])

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\rho_f\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon^2}\mathbf{q}\cdot\nabla\mathbf{q}\right] = -\nabla p - \frac{\mu_f}{K}\mathbf{q} + \mu_e\nabla^2\mathbf{q} + \rho_f\mathbf{g} \tag{2}$$

$$\varepsilon(\rho c)_f\frac{\partial T_f}{\partial t} + (\rho c)_f\mathbf{q}\cdot\nabla T_f = \varepsilon k_f\nabla^2 T_f + h(T_s - T_f) \tag{3}$$

$$(1-\varepsilon)(\rho c)_s\frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s - h(T_s - T_f) \tag{4}$$

$$\rho_f = \rho_0[1 - \beta(T_f - T_u)] \tag{5}$$

We note that ε very small refers to densely packed porous media and ε nearly one refers to sparsely packed porous media. One important thing that needs special attention is the idea that the fluid can be mechanically incompressible, but that its volume can change with temperature. This is at the centre of the approximation that was first studied by Oberbeck and later by Boussinesq, known as the Oberbeck–Boussinesq approximation. Recently, Rajagopal et al. [16] have given a sound mathematical basis for this approximation.

We confine ourselves to two-dimensional motions. Further, the boundaries are assumed to be free–free and isothermal. The basic state is assumed to be quiescent and we superimpose a small perturbation on it. Eqs. (1)–(5) are now made dimensionless using following transformations:

$$\begin{aligned} &(x,y) = d(x^*,y^*)\\ &(u,v) = \frac{\varepsilon k_f}{(\rho c)_f d}(u^*,v^*)\\ &p = \frac{k_f\mu}{(\rho c)_f K}p^*\\ &T_f = (T_l - T_u)\theta + T_u\\ &T_s = (T_l - T_u)\phi + T_u\\ &t = \frac{(\rho c)_f d^2}{k_f}t^* \end{aligned} \tag{6}$$

We eliminate the pressure from the momentum transport equation (2) and arrive at the vorticity transport equation. The non-dimensional form of the vorticity and the heat transport equations are obtained in the form

$$\frac{Da}{\varepsilon Pr}\left[\frac{\partial}{\partial t}\nabla^2\psi - J(\psi,\nabla^2\psi)\right] = Da\nabla^4\psi - \nabla^2\psi + R\frac{\partial\theta}{\partial x} \tag{7}$$

$$\frac{\partial\theta}{\partial t} - J(\psi,\theta) - \frac{\partial\psi}{\partial x} = \nabla^2\theta + H(\phi-\theta) \tag{8}$$

$$\alpha\frac{\partial\phi}{\partial t} = \nabla^2\phi + \gamma H(\theta - \phi) \tag{9}$$

where ψ is the stream function, which is related to u and v by

$$u = -\frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}$$

$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplacian operator and J is the Jacobian. The asterisks have been removed for simplicity.

The non-dimensional groups that appear in the above equations are

$$\begin{aligned} &R = \frac{\rho_0 g\beta(T_l - T_u)Kd}{\varepsilon\mu_f\kappa_f}\\ &\gamma = \frac{\varepsilon k_f}{(1-\varepsilon)k_s}, \quad H = \frac{hd^2}{\varepsilon k_f}\\ &\alpha = \frac{(\rho c)_s}{(\rho c)_f}\frac{k_f}{k_s} = \frac{\kappa_f}{\kappa_s} \quad Da = \frac{\mu_e}{\mu_f}\frac{K}{d^2}\\ &Pr = \frac{\mu_e(\rho c)_f}{\rho_0 k_f} \end{aligned} \tag{10}$$

In Eq. (10), R is the Darcy–Rayleigh number which expresses the balance between buoyancy and viscous forces, γ is the porosity modified conductivity ratio, H is the non-dimensional inter phase heat transfer coefficient, α is the diffusivity ratio, Da is the Darcy number and Pr is the Prandtl number.

We note that, the fluid and solid phases are not in thermal equilibrium, the use of appropriate boundary conditions for the temperature fields may pose a difficulty. However we assume that the phases have equal temperatures at the bounding surfaces. Therefore Eqs. (7)–(9) are solved for stress-free isothermal boundaries and hence the boundary conditions are

$$\psi = \frac{\partial^2\psi}{\partial y^2} = 0 \text{ on } y = 0 \text{ and } 1 \tag{11a}$$

$$\theta = \phi = 0 \text{ on } y = 0 \text{ and } 1 \tag{11b}$$

3. Linear stability theory

We assume that the equilibrium state is subjected to infinitesimal perturbations. To study the linear theory we use the linearized version of Eqs. (7)–(9). The principle of exchange of stability (PES) may be proved easily so that the onset of convection is stationary (see Appendix A for the proof of the validity of the PES).

We seek the solutions to the linearized equations in the form

$$\psi = A_1 \sin \pi y \cos kx \tag{12a}$$

$$\theta = A_2 \sin \pi y \sin kx \tag{12b}$$

$$\phi = A_3 \sin \pi y \sin kx \tag{12c}$$

where k is the horizontal wave number and the A 's are constants. Substitution of Eq. (12) into the linearized version of the Eqs. (7)–(9) yields the following matrix equation:

$$\begin{pmatrix} (\pi^2+k^2)[Da(\pi^2+k^2)+1] & Rk & 0\\ k & (k^2+\pi^2+H) & -H\\ 0 & \gamma H & -(k^2+\pi^2+\gamma H) \end{pmatrix}\begin{pmatrix} A_1\\ A_2\\ A_3\end{pmatrix} = \begin{pmatrix} 0\\0\\0\end{pmatrix} \tag{13}$$

By setting the determinant of the coefficient matrix to zero we get

$$R = \left(\frac{(\pi^2+k^2)^2}{k^2} + \frac{Da(\pi^2+k^2)^3}{k^2}\right)\left(1 + \frac{H}{k^2+\pi^2+\gamma H}\right) \tag{14}$$

For given values of H and γ Eq. (14) describes the neutral curves for the onset of convection. Further we note from Eq. (14) that Rayleigh number is of order (k^{-4}) as $k \to 0$ and it is of order (k^4) as $k \to \infty$. We display neutral curves for $H = 100$ and for a range of values of γ in Fig. 1(a)–(d). These figures show the transition from Brinkman regime to the Darcy regime ($Da \to 0$). We observe from these figures that, in general the Rayleigh number R decreases as γ increases. Thus the effect of increasing the conductivity ratio is to destabilize the system. The effect is more pronounced for small γ. The neutral curves are more close and branches are apart for different γ in case of Darcy regime while for Brinkman regime it is reverse.

The value of the Rayleigh number R given in Eq. (14) can be minimized with respect to the wave number k by setting $\frac{\partial R}{\partial k} = 0$ and solving this equation. However in the present case it is highly impossible to obtain a straightforward closed form explicit expression for the minimizing value of k. Therefore, we use Newton–Raphson iteration scheme to obtain the minimum values of R and k as function of H and γ.

In Fig. 2(a)–(d), we show the variation of the critical Rayleigh number R_c with H for a range of values of γ. We observe from these figures that the critical Rayleigh number is independent of γ for very small values of H while for large H the critical Rayleigh number decreases with increasing γ. For large γ ($\geqslant 10$) the critical Rayleigh number is independent of H. The interphase heat transfer coefficient H is very small means that there is almost no transfer of heat between the phases, and therefore the critical value is not affected by the properties of the solid phase. On the other hand for large values of H that is, in the LTE limit the critical value is based on the mean properties of the medium and R_c is dependent on γ. For fixed γ, R_c vary monotonically as H increases.

One important thing to note about the definition of the Rayleigh number is that it is based on the properties of the fluid. We now modify R and redefine it as

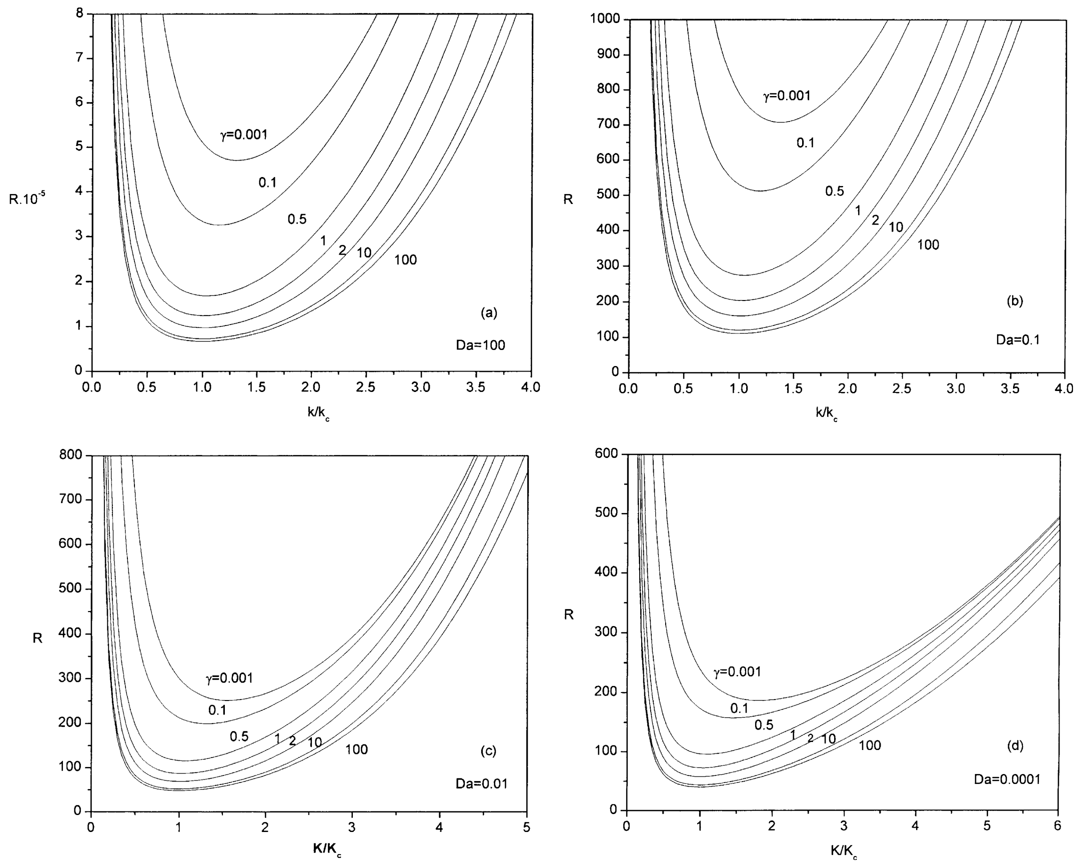

Fig. 1. Neutral curves for different values of γ, Da and $H = 100$.

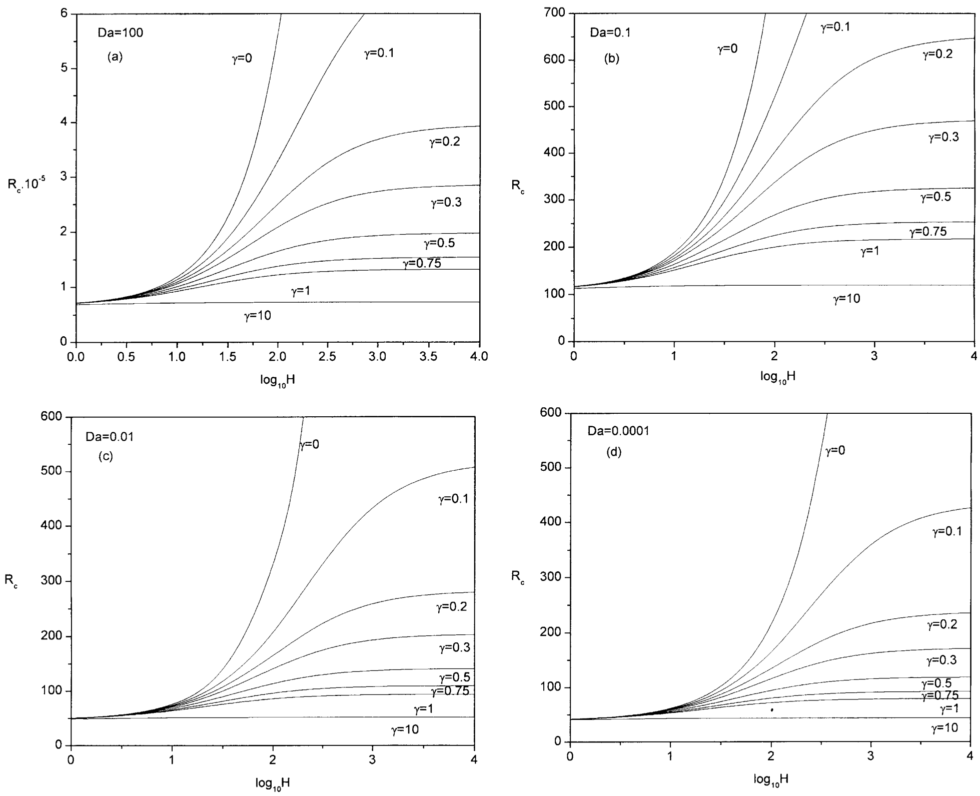

Fig. 2. Variation of critical Rayleigh number R_c with H.

$$R_{MP} = \left(\frac{\gamma}{1+\gamma}\right)R = \frac{\rho_f g\beta(T_l - T_u)Kd}{[\varepsilon\kappa_f + (1-\varepsilon)\kappa_s]\mu_f} \tag{15}$$

which is now based on the mean properties of the porous medium. In fact it is this value, which is used in the local thermal equilibrium (LTE) case.

In Fig. 3(a)–(d), we show the variation of R_{MP}, the critical Rayleigh number which is defined in terms of the mean properties of the fluid with the inter phase heat transfer coefficient H for specific values of γ. It is interesting to note that R_{MP} approaches a common limit as $H \to \infty$, however the approach to the common limit is different for different values of γ. We also find that R_{MP} vary monotonically as H increases. It is interesting to note that, for very small H, and large γ, the convection can be completely suppressed. The critical Rayleigh number based on the mean properties is independent of H for large γ.

The variation of the critical wave number k_c with H for different values of the conductivity ratio γ is shown in Fig. 4(a)–(d). We observe that the critical wave number k_c approaches a common limit for small Da as $H \to 0$ and $H \to \infty$. However for large Da, k_c approaches two different limits, one as $H \to 0$ and another as $H \to \infty$. For very small H the solid phase ceases to affect the thermal field of the fluid. On the other hand as $H \to \infty$, the solid and fluid phase have almost equal temperatures. Therefore in these two limiting cases, the conductivity ratio has little effect on the critical wave number. For the intermediate values of H, the critical wave number k_c increases with decreasing values of γ and that the critical wave number is always greater than the LTE case.

4. Asymptotic analysis

Case 1: For very small values of H

When H is very small the critical value of the Rayleigh number R is slightly above the critical value for the LTE case. Accordingly we expand R given by Eq. (14) in a power series in H as

$$R = \left(\frac{(\pi^2+k^2)^2}{k^2} + \frac{Da(\pi^2+k^2)^3}{k^2}\right) + \left(\frac{(\pi^2+k^2)}{k^2} + \frac{Da(\pi^2+k^2)^2}{k^2}\right)H - \left(\frac{1}{k^2} + \frac{Da(\pi^2+k^2)}{k^2}\right)\gamma H^2 + \cdots \tag{16}$$

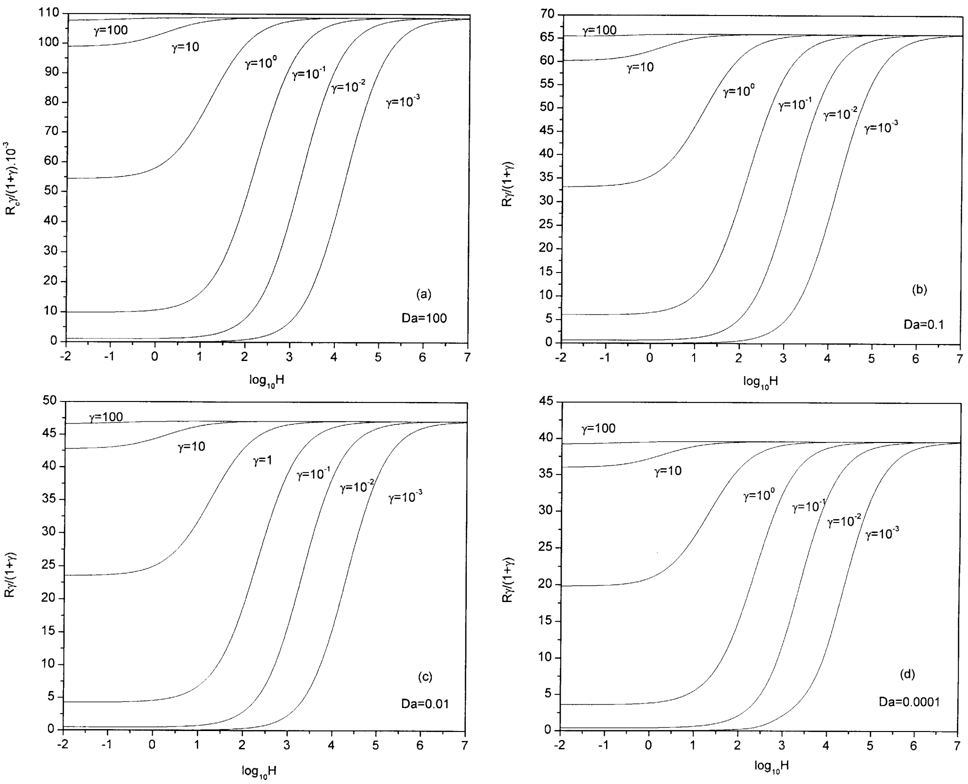

Fig. 3. Variation of critical Rayleigh number with H.

To minimize R up to $O(H^2)$ we set $\partial R/\partial k = 0$ and obtain an expression of the form

$$\begin{aligned}&(4Dak^6 + (2 + 6Da\pi^2)k^4 - (2\pi^4 + 2Da\pi^6))\\&\quad + (2Dak^4 - 2\pi^2 - 2Da\pi^4)H + (2 + 2Da\pi^2)H^2\gamma\\&\quad = 0\end{aligned} \tag{17}$$

We also expand k in power series of H as

$$k = k_0 + k_1 H + k_2 H^2 + \cdots \tag{18}$$

where k_0 is the critical wave number for the LTE case and is given by the expression

$$k_0^2 = \frac{-(Da\pi^2 + 1) + \sqrt{(Da\pi^2 + 1)(9Da\pi^2 + 1)}}{4Da} \tag{19}$$

Substituting Eq. (18) into Eq. (17) and rearranging the terms and then equating the coefficients of same powers of H will allow us to find k_1 and k_2. Thus we obtain

$$k_1 = \frac{\Delta_1}{\Delta} \tag{20}$$

$$k_2 = \frac{\Delta_2}{\Delta} \tag{21}$$

where

$$\Delta_1 = \pi^2 + Da\pi^4 - Dak_0^4$$

$$\begin{aligned}\Delta_2 &= (2 + 2Da\pi^2)\gamma + (60Dak_0^4 + 12k_0^2 + 36Dak_0^2\pi^2)k_1^2\\&\quad + 8Dak_0^3 k_1\end{aligned}$$

$$\Delta = 12Dak_0^5 + 4k_0^3 Da + 12Da\pi^2 k_0^3$$

With these values of k_0, k_1 and k_2, Eq. (18) gives the critical wave number and consequently using this in (16) one can obtain the critical Rayleigh number for small H.

Case 2: For very large values of H

For large values H, the critical Rayleigh number takes the form

$$\begin{aligned}R_c &= \left(\frac{(\pi^2 + k^2)^2}{k^2} + Da\frac{(\pi^2 + k^2)^3}{k^2}\right)\\&\quad \times \left(\frac{1+\gamma}{\gamma} - \frac{(\pi^2 + k^2)}{\gamma^2}H^{-1} + \frac{(\pi^2 + k^2)^2}{\gamma^3}H^{-2}\right)\end{aligned} \tag{22}$$

We minimize this with respect to k in a similar way as we did in the small H case and obtain the following expression:

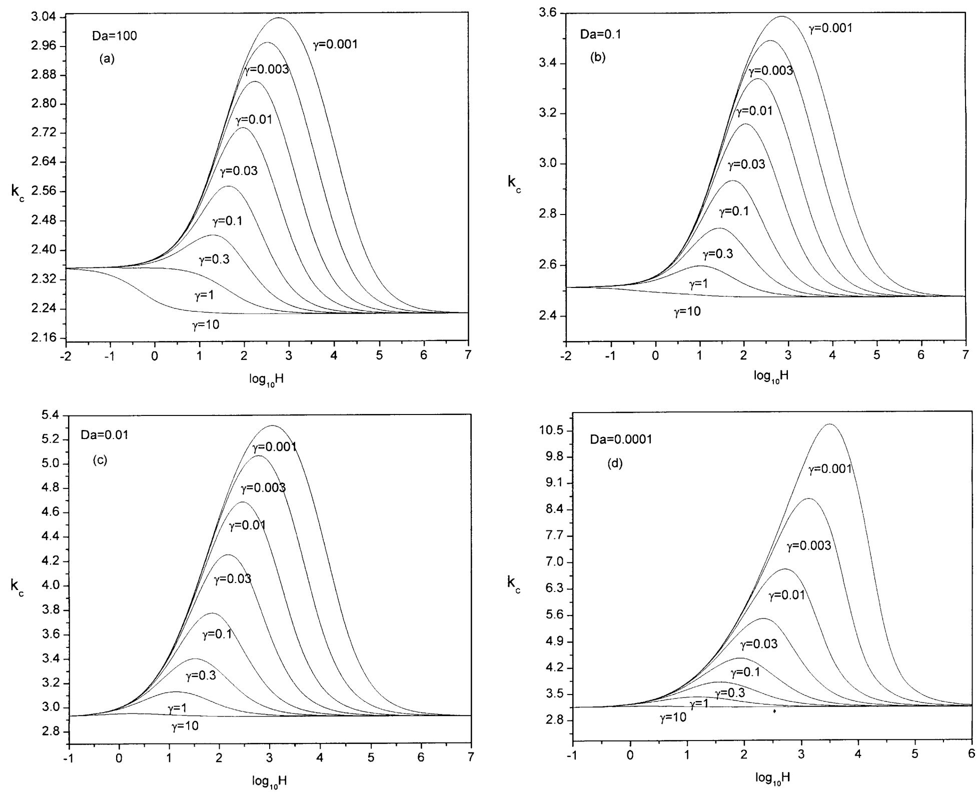

Fig. 4. Variation of critical wave number with H.

$$
\begin{aligned}
&\big(8Dak^{10}+6(5Da\pi^2+1)k^8+4(10Da\pi^4+4\pi^2)k^6\\
&\quad+2(10Da\pi^6+6\pi^4)k^4-2\pi^8-2Da\pi^{10}\big)H^{-2}\\
&\quad-\big(6\gamma Dak^8+4\gamma(Da\pi^2+3Da\pi^2\gamma)k^6\\
&\quad+2\gamma\pi^2(3+2Da\pi^2)k^4-2\gamma\pi^6(1+Da\pi^2)\big)H^{-1}\\
&\quad+4Da(\gamma^2+\gamma^3)k^6+2(1+3Da\pi^2)(\gamma^2+\gamma^3)k^4\\
&\quad-\pi^4-Da\pi^6=0
\end{aligned}
\tag{23}
$$

Similarly we expand k in the form

$$k=k_0+\frac{k_1}{H}+\frac{k_2}{H^2}+\cdots \tag{24}$$

where k_0 is given by Eq. (19) and k_1, k_2 are to be found.

Substituting Eq. (24) into Eq. (23) and equating the coefficients of like powers of H we can find k_1 and k_2 and are given by

$$k_1=\frac{\Delta_1'}{\Delta'} \tag{25}$$

$$k_2=\frac{\Delta_2'}{\Delta'} \tag{26}$$

where

$$
\begin{aligned}
\Delta_1'&=6Da\gamma k_0^8+4\gamma(Da\pi^2+3Da\gamma\pi^2)k_0^6\\
&\quad+2\gamma(3\pi^2+6Da\pi^4)k_0^4-2\gamma\pi^2(\pi^4+Da\pi^6)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_2'&=\big(48Da\gamma k_0^7+24\gamma(Da\pi^2+3Da\gamma\pi^2)k_0^5\\
&\quad+8\gamma(3\pi^2+6Da\pi^4)k_0^3\big)k_1\\
&\quad-\big((8Dak_0^{10}+6(5Da\pi^2+1)k_0^8+4(10Da\pi^4+4\pi^2)k_0^6\\
&\quad+2(10Da\pi^6+6\pi^4)k_0^4-2\pi^8-2Da\pi^{10})\\
&\quad-(60Da(\gamma^2+\gamma^3)k_0^4+12(1+3Da\pi^2)(\gamma^2+\gamma^3)k_0^2)\big)k_1^2
\end{aligned}
$$

$$\Delta'=24Da(\gamma^2+\gamma^3)k_0^5+8(1+3Da\pi^2)(\gamma^2+\gamma^3)k_0^3$$

Again with these values of k_0, k_1 and k_2, we compute the critical wave number k_c from Eq. (24) and finally using this value of k_c, one can obtain the critical Rayleigh number R_c from Eq. (22) for large H.

The expression for the critical Rayleigh number R_c and the critical wave number k_c for both small H and large H are evaluated for fixed value of $Da = 100$ and comparison of these values with the exact values obtained earlier are given in Tables 1 and 2. It is important to note that an excellent agreement between these two results are found when H is small. On the other hand reasonably good agreement is found when H is large.

Table 1
Comparison of Exact and Asymptotic solutions for small values of H

$\text{Log}_{10} H$	k_c(E)	k_c(A)	R_c(E)	R_c(A)
$\gamma = 1$				
−2.0	2.223085	2.223085	65945.95	65945.95
−1.5	2.223890	2.223894	66041.82	66041.82
−1.0	2.226392	2.226431	66343.04	66343.04
−0.5	2.233898	2.234266	67276.30	67276.30
0.0	2.254083	2.257160	70049.47	70049.47
0.5	2.293710	2.310735	77391.09	77391.09
$\gamma = 0.01$				
−2.0	2.223085	2.223086	65945.98	65945.98
−1.5	2.223895	2.223899	66042.13	66042.13
−1.0	2.226441	2.226481	66345.99	66343.98
−0.5	2.234371	2.234767	67305.19	67305.20
0.0	2.258288	2.262172	70322.34	70322.62
0.5	2.324203	2.360850	79725.57	79751.59

Table 2
Comparison of Exact and Asymptotic solutions for large values of H

$\text{Log}_{10} H$	k_c(E)	k_c(A)	R_c(E)	R_c(A)
$\gamma = 1$				
2.5	2.237412	2.237384	129390.20	129390.80
3.0	2.229131	2.229130	131395.20	131395.50
3.5	2.226373	2.226373	132055.00	132055.00
4.0	2.225486	2.225486	132266.30	132266.30
4.5	2.225204	2.225204	132333.40	132333.40
5.0	2.225115	2.225115	132354.60	132354.60
5.5	2.225087	2.225087	132361.40	132361.40
6.0	2.225078	2.225078	132363.50	132363.50
$\gamma = 0.01$				
4.5	2.250385	2.237865	6386385.00	6387308.00
5.0	2.233200	2.229487	6587409.00	6587455.00
5.5	2.227656	2.226506	6653446.00	6653450.00
6.0	2.225892	2.225530	6674586.00	6674587.00
6.5	2.225332	2.225219	6681298.00	6681298.00
7.0	2.225156	2.225120	6683422.00	6683423.00
7.5	2.225100	2.225088	6684095.00	6684095.00
8.0	2.225082	2.225078	6684308.00	6684307.00

Note. E denotes exact and A denotes asymptotic.

5. Conclusion

The stability of a horizontal fluid saturated sparsely packed porous layer heated from below and cooled form above when the solid and fluid phases are not in local thermal equilibrium is examined analytically. Brinkman model with inertia term is used for the momentum equation and a two-field model is used for energy equation each representing the solid and fluid phases separately. Although the inertia term is included in the general formulation, it does not affect the stability condition since the basic state is motionless. The condition for the onset of convection is obtained analytically. We display the results in Figs. 1–4 for a wide range of values of γ and H and for $Da = 10^2$, 10^{-1}, 10^{-2} and 10^{-3} that show the transition from Brinkman regime to the Darcy regime.

The effect of increasing conductivity ratio γ is to decrease the critical Rayleigh number and hence the effect of increasing γ is to destabilize the system. The effect is more pronounced for very small γ. The critical Rayleigh number is independent of γ for very small H while for large H, it decreases with increasing γ.

It is found that the critical Rayleigh number R_{MP} based on the mean properties of the media vary monotonically with H and approaches a common limit as $H \to \infty$. It is also observed that the critical wave number k_c approaches a common limit as $H \to 0$ and $H \to \infty$ in the Darcy limit while it approaches two different limits, one as $H \to 0$ and another as $H \to \infty$ in the Brinkman regime. An excellent agreement is found between the exact solutions and the solutions obtained from asymptotic analysis. The results of the LTE case are recovered in the large H limit and the results of Darcy regime are recovered in the small Da limit.

Acknowledgments

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Appendix A. Eqs. (7)–(9), on neglecting nonlinear terms, become

$$\left(1 + \frac{Da}{\varepsilon Pr}\frac{\partial}{\partial t}\right)\nabla^2\psi - Da\nabla^4\psi - R\frac{\partial\theta}{\partial x} = 0 \tag{A.1}$$

$$\frac{\partial\theta}{\partial t} - \nabla^2\theta - H(\phi - \theta) - \frac{\partial\psi}{\partial x} = 0 \tag{A.2}$$

$$\alpha\frac{\partial\phi}{\partial t} - \nabla^2\phi - \gamma H(\theta - \phi) = 0 \tag{A.3}$$

Assume the solution of Eqs. (A.1)–(A.3) in the form

$$\begin{aligned} \psi &= e^{\sigma t}\Psi(y)\cos kx \\ \theta &= e^{\sigma t}\Theta(y)\sin kx \\ \phi &= e^{\sigma t}\Phi(y)\sin kx \end{aligned} \tag{A.4}$$

where σ is the growth rate, k is the horizontal wave number and $\Psi(y)$, $\Theta(y)$ and $\Phi(y)$ are the amplitudes of perturbation stream function, perturbation fluid temperature and perturbation solid temperature respectively.

Substituting Eq. (A.4) into Eqs. (A.1)–(A.3), we get

$$\left(1+\frac{Da}{\varepsilon Pr}\sigma\right)(\mathrm{D}^2-k^2)\Psi - Da(\mathrm{D}^2-k^2)^2\Psi - Rk\Theta = 0 \tag{A.5}$$

$$\sigma\Theta - (\mathrm{D}^2-k^2)\Theta - H(\Phi-\Theta) - k\Psi = 0 \tag{A.6}$$

$$\alpha\sigma\Phi - (\mathrm{D}^2-k^2)\Phi - \gamma H(\Theta-\Phi) = 0 \tag{A.7}$$

where $\mathrm{D}=\frac{\mathrm{d}}{\mathrm{d}y}$.

Multiplying Eq. (A.5) by Ψ^* the complex conjugate of Ψ, integrating between y = 0 and 1 and using the boundary conditions yields

$$\left(1+\frac{Da}{\varepsilon Pr}\sigma\right)\langle|\mathrm{D}\Psi|^2+k^2|\Psi|^2\rangle + Da\langle|\mathrm{D}^2\Psi|^2+2k^2|\mathrm{D}\Psi|^2 + k^4|\Psi|^2\rangle + Rk\langle\Psi^*\Theta\rangle = 0 \tag{A.8}$$

where $\langle\cdots\rangle=\int_0^1(\cdots)\mathrm{d}y$ and $*$ indicate that the quantity is the complex conjugate. Taking complex conjugate of Eq. (A.6) and multiplying the resulting equation by $\Theta$and integrating between $y = 0$ and 1 and using the boundary conditions, we obtain

$$k\langle\Psi^*\Theta\rangle = -\sigma^*\langle|\Theta|^2\rangle - \langle|\mathrm{D}\Theta|^2+|k\Theta|^2\rangle + H\langle\Phi^*\Theta\rangle - H\langle|\Theta|^2\rangle \tag{A.9}$$

Using Eq. (A.9) in Eq. (A.8) we get

$$\left(1+\frac{Da}{\varepsilon Pr}\sigma\right)\langle|\mathrm{D}\Psi|^2+k^2|\psi|^2\rangle + Da\langle|\mathrm{D}^2\Psi|^2+2k^2|\mathrm{D}\Psi|^2 + k^4|\Psi|^2\rangle - R\sigma^*\langle|\Theta|^2\rangle - R\langle|\mathrm{D}\Theta|^2+k^2|\Theta|^2\rangle + RH\langle\Phi^*\Theta\rangle - RH\langle|\Theta|^2\rangle = 0 \tag{A.10}$$

Multiplying Eq. (A.7) by Φ^*, integrating between $y = 0$ and 1 and using the boundary conditions we obtain

$$H\langle\Phi^*\Theta\rangle = \frac{\alpha}{\gamma}\sigma\langle|\Phi|^2\rangle + \frac{1}{\gamma}\langle|\mathrm{D}\Phi|^2+k^2|\Phi|^2\rangle + H\langle|\Phi|^2\rangle \tag{A.11}$$

On using Eq. (A.11) in Eq. (A.10) we get

$$\left(1+\frac{Da}{\varepsilon Pr}\sigma\right)\langle|\mathrm{D}\Psi|^2+k^2|\Psi|^2\rangle + Da\langle|\mathrm{D}^2\Psi|^2+2k^2|\mathrm{D}\Psi|^2 + k^4|\Psi|^2\rangle - R\sigma^*\langle|\Theta|^2\rangle - R\langle|\mathrm{D}\Theta|^2+k^2|\Theta|^2\rangle + R\left(\frac{\alpha}{\gamma}\sigma\langle|\Phi|^2\rangle + \frac{1}{\gamma}\langle|\mathrm{D}\Phi|^2+k^2|\Phi|^2\rangle + H\langle|\Phi|^2\rangle\right) - RH\langle|\Theta|^2\rangle = 0 \tag{A.12}$$

Writing $\sigma = \mathrm{Re}(\sigma) + \mathrm{Im}(\sigma) = \sigma_1 + \mathrm{i}\sigma_2$ in Eq. (A.12) and equating real and imaginary parts we get

$$\mathrm{Im}(\sigma)\left[\frac{Da}{\varepsilon Pr}\langle|\mathrm{D}\Psi|^2+k^2|\Psi|^2\rangle + R\langle|\Theta|^2\rangle + \frac{R\alpha}{\gamma}\langle|\Phi|^2\rangle\right] = 0 \tag{A.13}$$

Since the quantity within the brackets is positive definite we must have $\mathrm{Im}(\sigma) = 0$. Hence the principle of exchange of stability is proved.

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